

Design of compression member:

(i) Column:

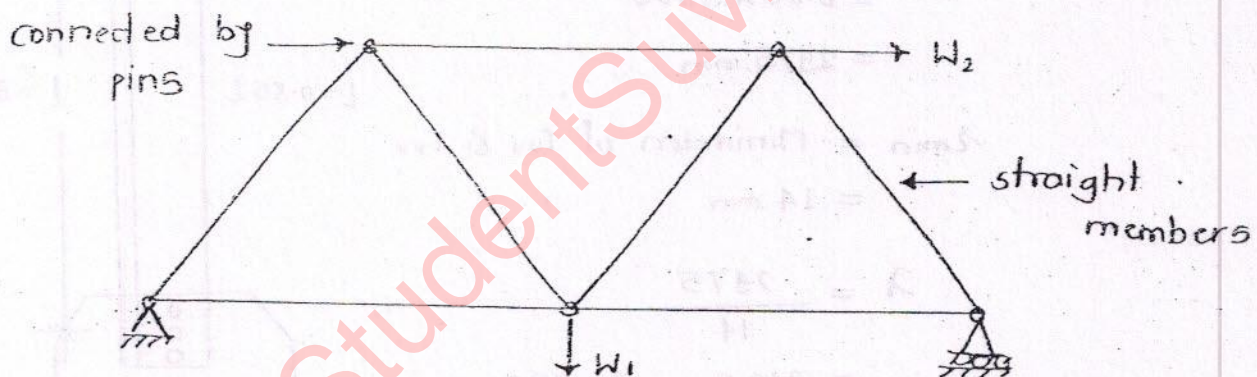
It is a structural member mainly subjected to compression. Bending moment also can exist in this member. This term 'column' is used for compression members of frames i.e. R.C.C. frames or steel frames.

(ii) Strut:

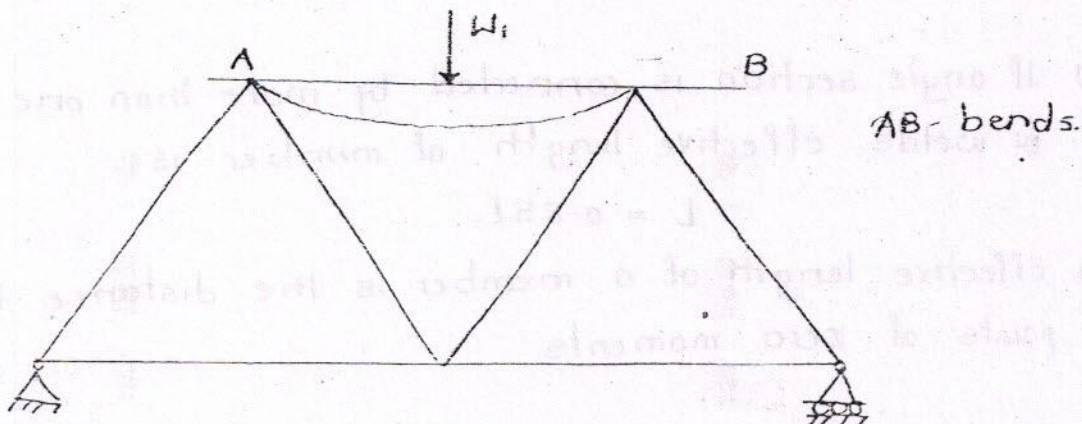
It is a structural member subjected to compression only. Bending moment is zero in this member. This term is used for compression members of trusses.

(iii) Truss:

It is a structure in which all the members are subjected to either tension or compression only. Bending moment is zero everywhere in the truss.



Truss (loads are applied only at joints)



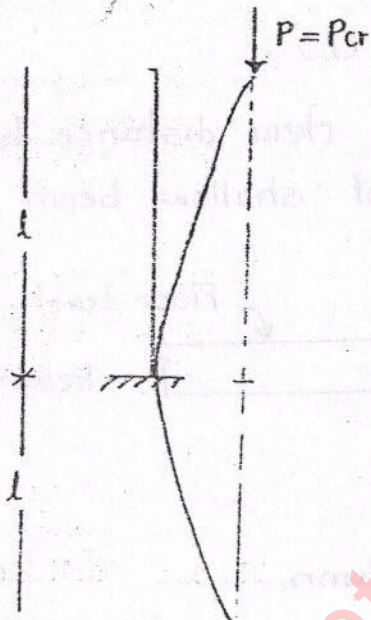
Frame (loads are applied at members)

(iv) Frames :

It is structure which is subjected to bending moment also in addition to tension and compression.

(v) Maximum slenderness ratio for compression members is 180 (less value because of buckling possibility in a compression members).

(vi) Effective length of columns:



$$L = 2l$$

- Theoretical and Recommended value.

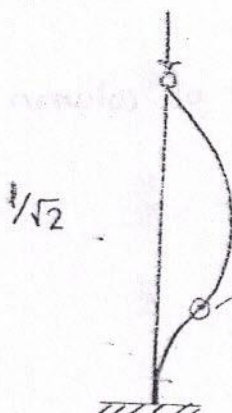
$$P_{cr} = \frac{\pi^2 EI}{4l^2}$$



$$L = l$$

- Theoretical and recommended value

$$P_{cr} = \frac{\pi^2 EI}{l^2}$$



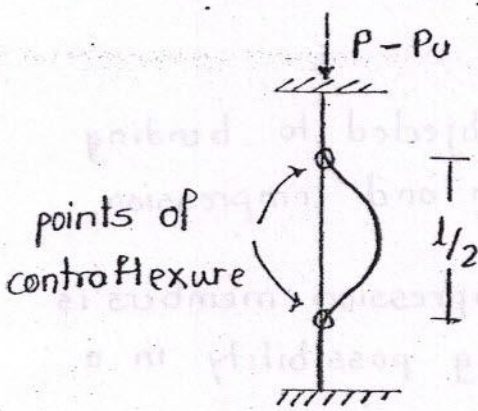
$$L = 0.707l$$

(Theoretical value from S.F.1.)

$$L = 0.8l$$

(Recommended value by IS 800)

(High value recommended because we can not maintain 100% fixity at fixed end)



$$L = 0.5 L$$

(Theoretical value)

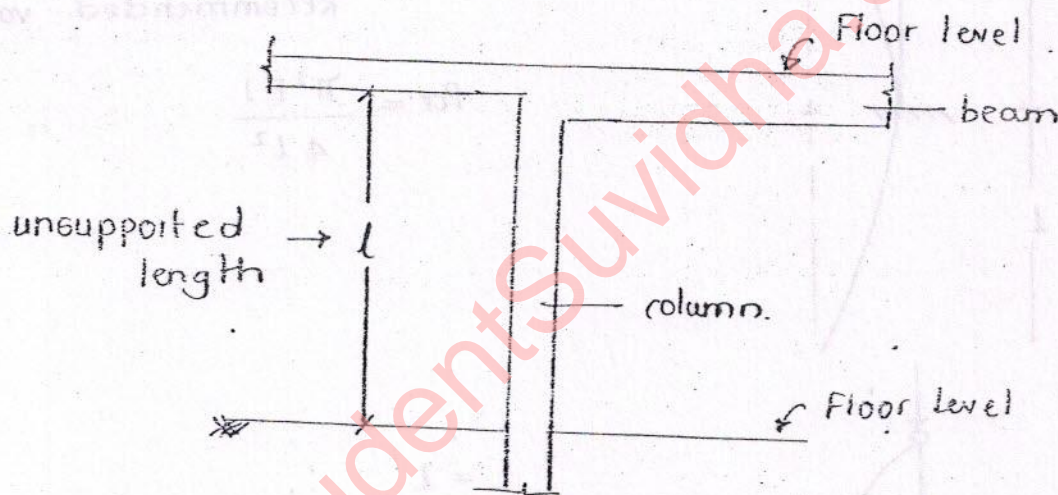
$$L = 0.65 L$$

(Recommended by IS 800)

Note:

(i) Unsupported length of column (L):

It is the maximum clear distance between top of the floor and bottom of shallow beam as shown in fig.



(ii) Effective length of column (L)

It is the length of column between points of zero moments.

(iii) Slenderness ratio (λ)

It is ratio of effective length of column and its minimum radius of gyration.

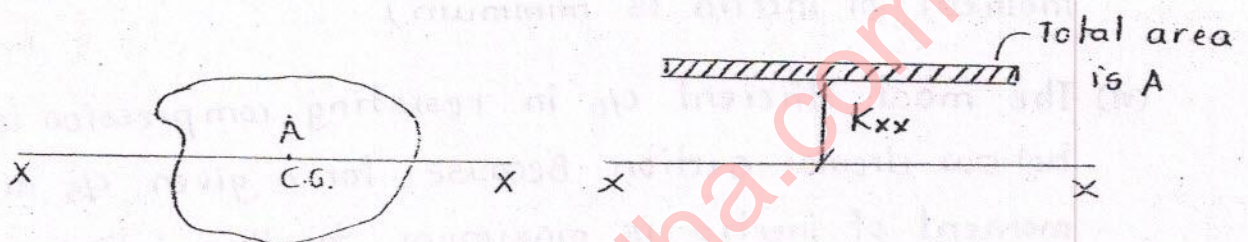
$$\lambda = \frac{L}{r_{min}}$$

(iv) Radius of gyration (r)

Gyration = revolution.

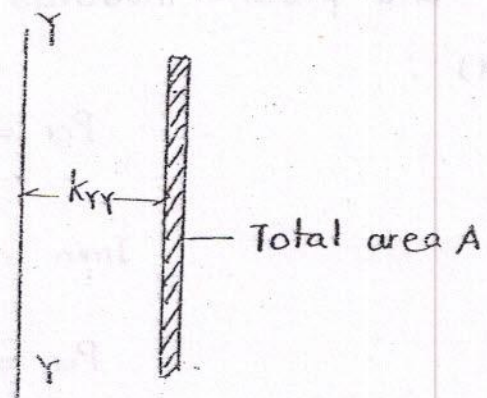
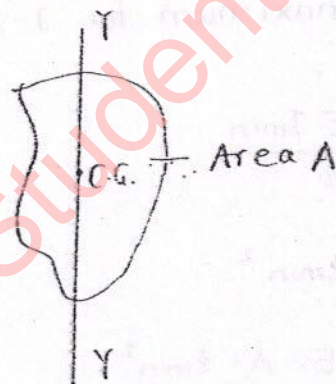
It is the distance at which entire area must be kept as a strip, so that it will give same moment of inertia as that of original area (about C.G.).

It is a measure of resistance to rotation or buckling. If radius of gyration is more, then it means that it is difficult to rotate or buckle the member about that axis.



$$I_{xx} = A \cdot k_{xx}^2 \quad (\text{2nd moment of area})$$

$$k_{xx} = \sqrt{\frac{I_{xx}}{A}}$$



$$k_{yy} = \sqrt{\frac{I_{yy}}{A}}$$

(v) The load carrying capacity of a column with one end fixed other end free is P . If the free end is made fixed then its load carrying capacity will be $16P$.

$$P = \frac{\pi^2 EI}{4l^2}$$

$$P_1 = \frac{4\pi^2 EI}{l^2}$$

$$\frac{\pi^2 EI}{4l^2} \times 16 = 16P$$

So load carrying capacity of the column depends on end condition and strongest column is both ends fixed column.

(vi) Euler's critical load :

$$P_{cr} = \frac{\pi^2 E I_{min}}{L^2}$$

I_{min} is taken always because columns tend to buckle about their weak axis (axis about which moment of inertia is minimum)

(vii) The most efficient c/s in resisting compression is thin hollow circular section. Because, for a given c/s area moment of inertia is maximum for thin hollow section so load carrying capacity is maximum.

(viii) The most efficient c/s in resisting bending moment is J-section. Because, for a given c/s area, section modulus and plastic modulus are maximum for J-section.

(ix) \therefore

$$P_{cr} = \frac{\pi^2 E I_{min}}{L^2}$$

$$I_{min} = A \cdot \bar{r}_{min}^2$$

$$P_{cr} = \frac{\pi^2 \cdot E \cdot A \cdot \bar{r}_{min}^2}{L^2}$$

$$P_{cr} = \frac{\pi^2 \cdot E \cdot A}{\left(\frac{L}{\bar{r}_{min}}\right)^2}$$

\therefore Euler's critical stress,

$$f_{ce} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2}$$

So, load carrying capacity of a long column, depends on flexural rigidity EI , and the axial rigidity AE and slenderness ratio ($\lambda = L/r$)

$$\frac{M}{I} = \frac{E}{R}$$

Flexural rigidity, $EI = M \times R \rightarrow 1 \text{ unit}$

It is the moment required to produce the unit radius of curvature in the member.

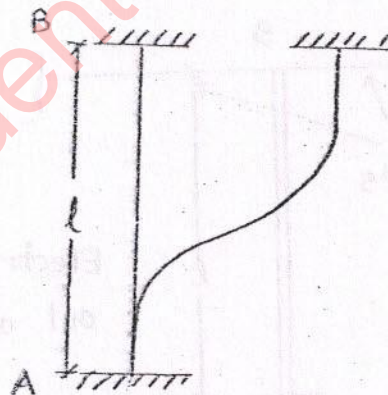
$$\delta l = \frac{PL}{AE}$$

Axial rigidity, $AE = \frac{PL}{\delta l} \rightarrow 1 \text{ unit}$

It is the force required to produce unit elongation in the body of unit length.

Note: :

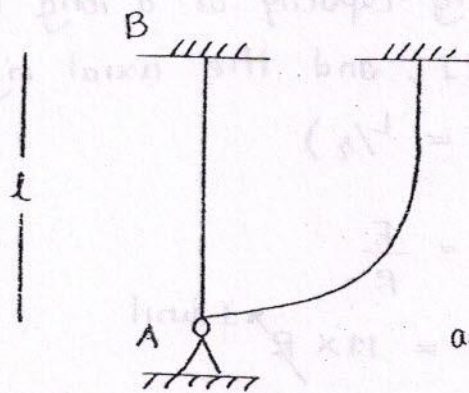
(i)



End A is effectively held in position and restrained against rotation.

End B is restrained against rotation but not held in position effectively (i.e., movable fixed support)

(i)

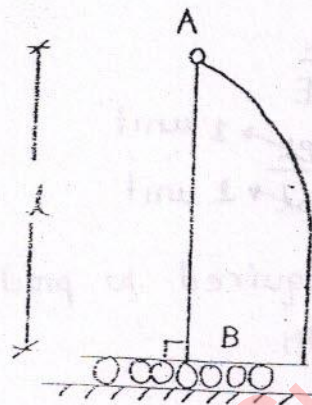


$$L = 2l$$

(Deflection profile is same as cantilever column)

End A is hinged support while end B is movable fixed support.

(ii)

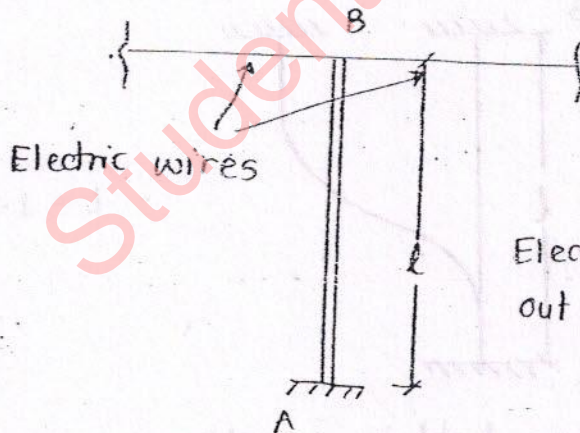


$$L = 2l$$

(Deflection is same as cantilever column)

Rigid plate.

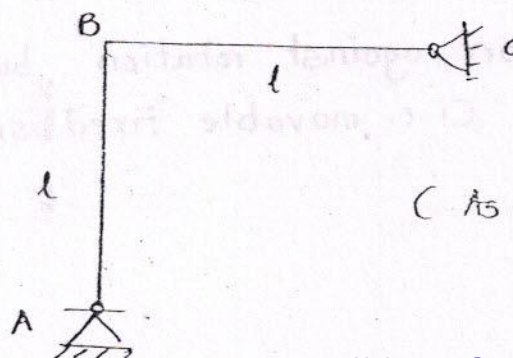
(iii)



$$L = 2l$$

Electric pole (free to buckle out of plane of wires)

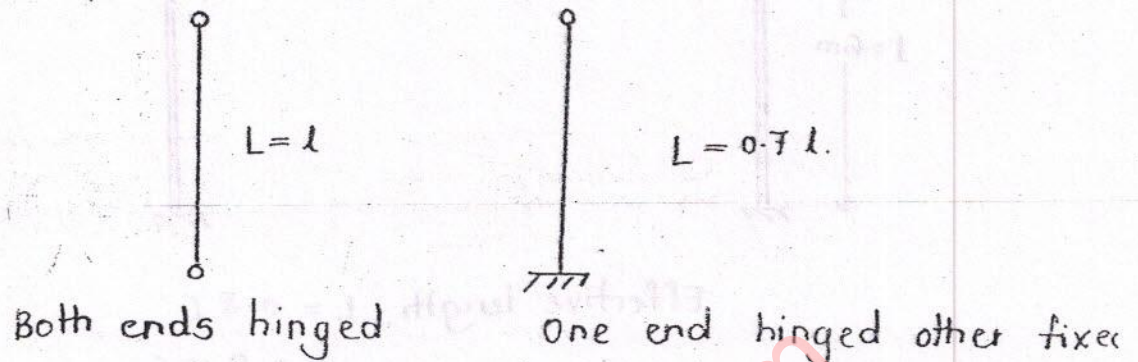
Q. What is effective length L for



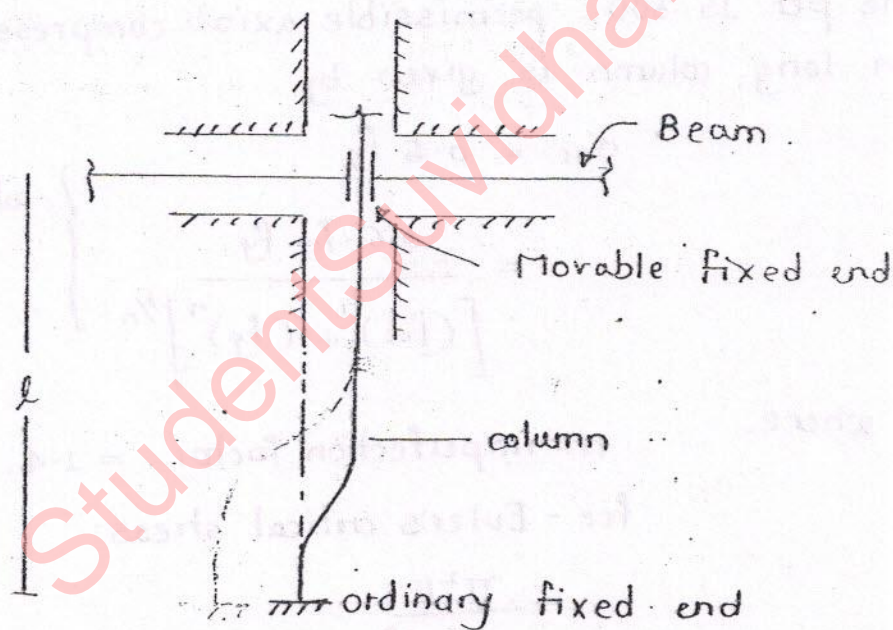
$$L = 0.7 l \text{ to } l$$

(As B is neither hinged nor fixed)

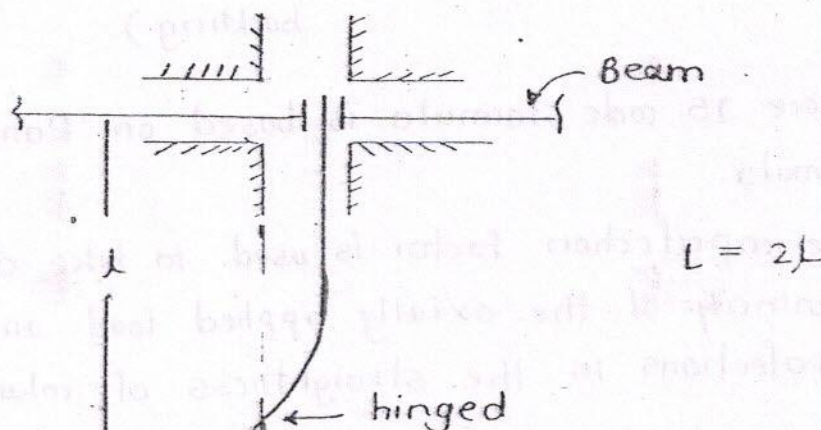
Rigid joint does not behaves like pin joint or fixed support. It requires some moment to rotate so. th effective length lies between $0.7L$ to L

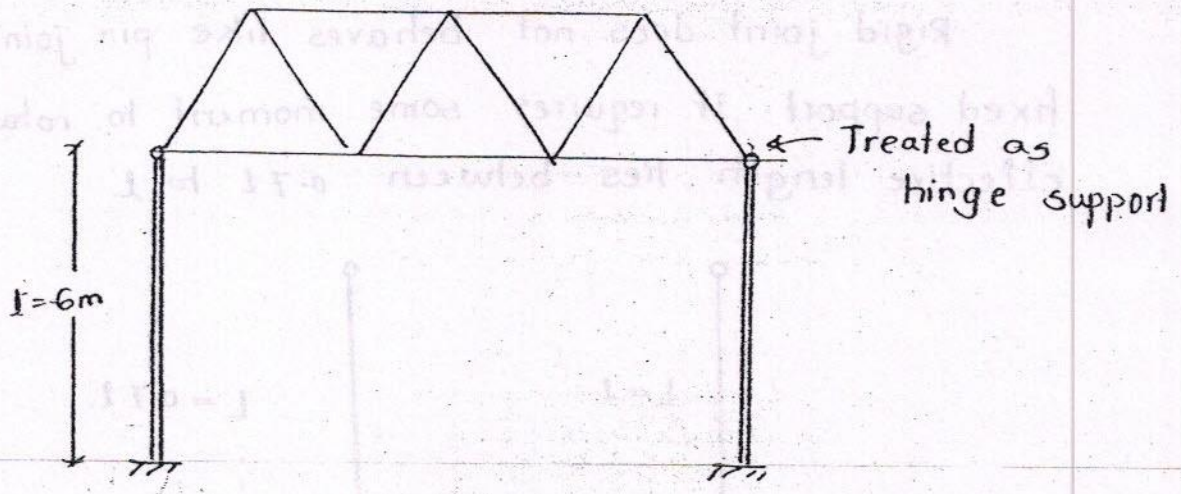


Q. What is effective length 'L'



Effective length = $1.2L$





$$\begin{aligned}\text{Effective length, } L &= 0.8 L \\ &= 0.8 \times 6 \\ &= 4.8 \text{ m}\end{aligned}$$

Note:

- (i) As per IS 800 : permissible axial compressive stress in long column is given by.

$$\begin{aligned}\sigma_{ac} &= 0.6 f_y \\ &= \frac{0.6 \cdot f_{cc} \cdot f_y}{\left[(f_{cc})^2 + (f_y)^2 \right]^{1/2}} \end{aligned} \quad \left. \vphantom{\begin{aligned} \sigma_{ac} &= 0.6 f_y \\ &= \frac{0.6 \cdot f_{cc} \cdot f_y}{\left[(f_{cc})^2 + (f_y)^2 \right]^{1/2}} \end{aligned}} \right\} \begin{array}{l} \text{whichever is} \\ \text{less.} \end{array}$$

where,

n - imperfection factor = 1.4.

f_{cc} - Euler's critical stress

$$= \frac{\pi^2 E}{\left(\frac{L}{r} \right)^2}$$

(f_{cc} will take care of effect of buckling)

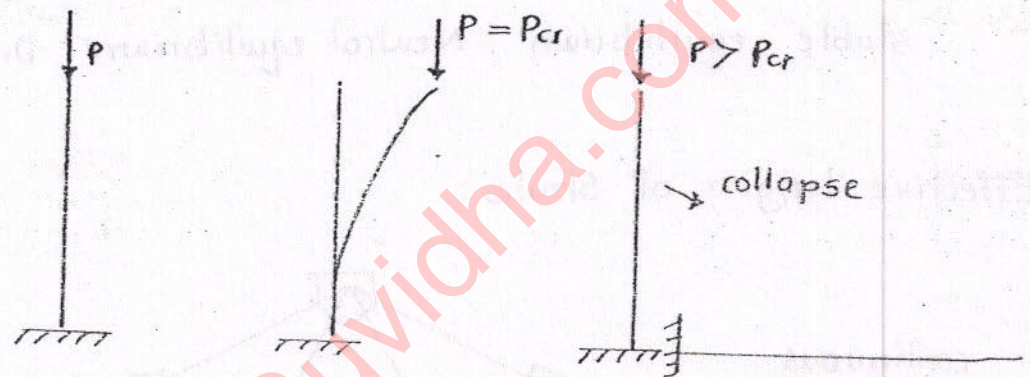
Above IS code formula is based on 'Rankine-Machand' formula.

The imperfection factor is used to take care of the eccentricity of the axially applied load. and for the imperfections in the straightness of column.

(ii) In a long column, permissible axial compressive stress depends upon.

- length of column (L)
- end conditions (i.e. fixed, hinged and free)
- area of c/s (A)
- distribution of area (I)
- Young's modulus.

(iii) Euler's column formula is derived based on neutral equilibrium of column.



(iv) Types of equilibrium:

(a) Stable equilibrium:

If a body is displaced from its equilibrium position and if it comes back to original position then it is called stable equilibrium.

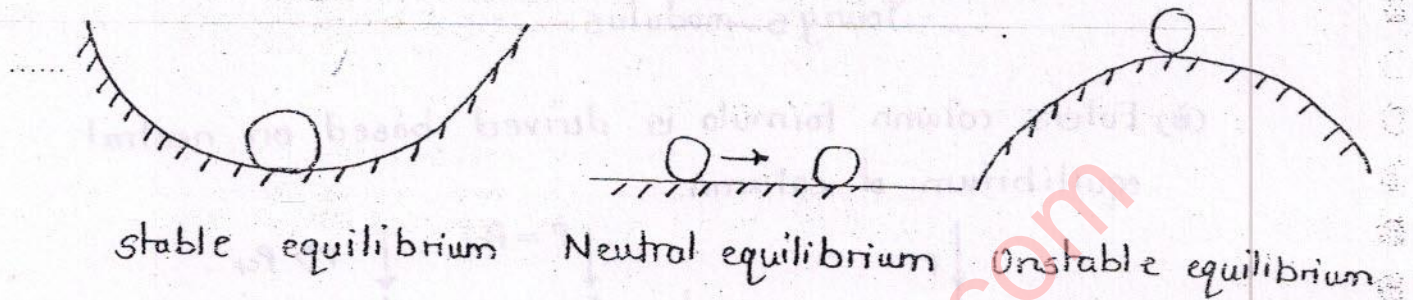
(b) Neutral equilibrium:

If a body is displaced from its equilibrium position and if it takes up another equilibrium position then it is called Neutral equilibrium.

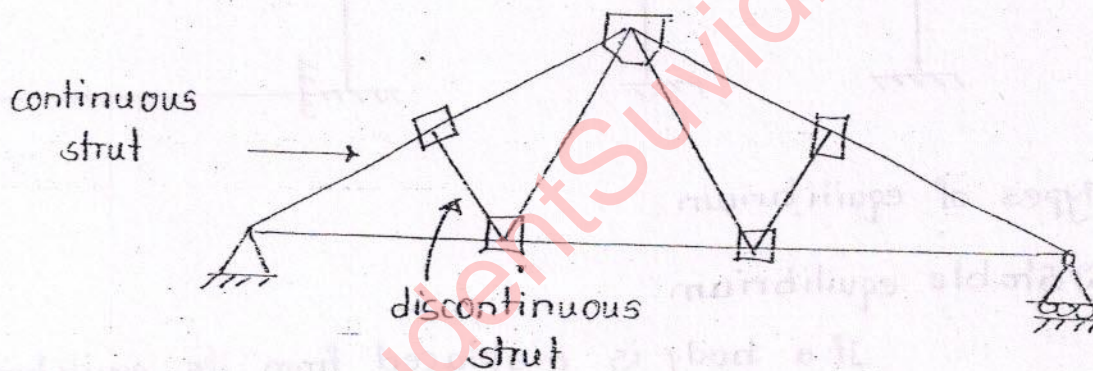
Neutral equilibrium is used to derive the Euler's formula.

Ⓒ Unstable equilibrium:

If a body is displaced from its equilibrium position, then if it does not come back to its original position nor takes up another equilibrium position then it is called Unstable equilibrium.



Effective length of struts:



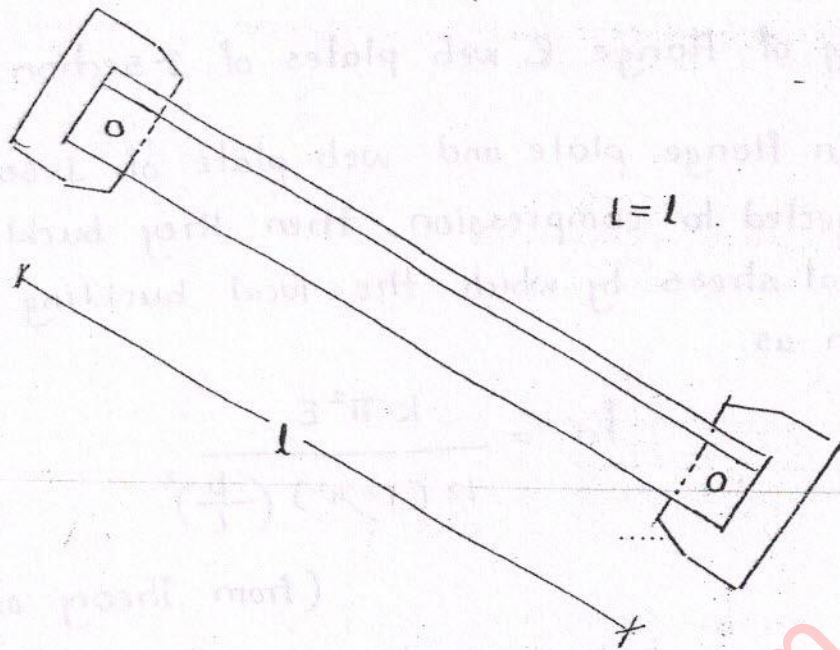
- (i) If a strut spans between two joints only, it is called a discontinuous strut. If it spans over more than two joints, then it is called a continuous strut.
- (ii) If a single angle discontinuous strut is connected by only one rivet at each end, then

(a) Effective length,

$$L = l$$

(b) Axial compressive stress,

$$\sigma_{ac'} = 0.8 \sigma_{ac} \quad (\text{permissible } \sigma_{ac} \text{ is reduced by } 20\%)$$



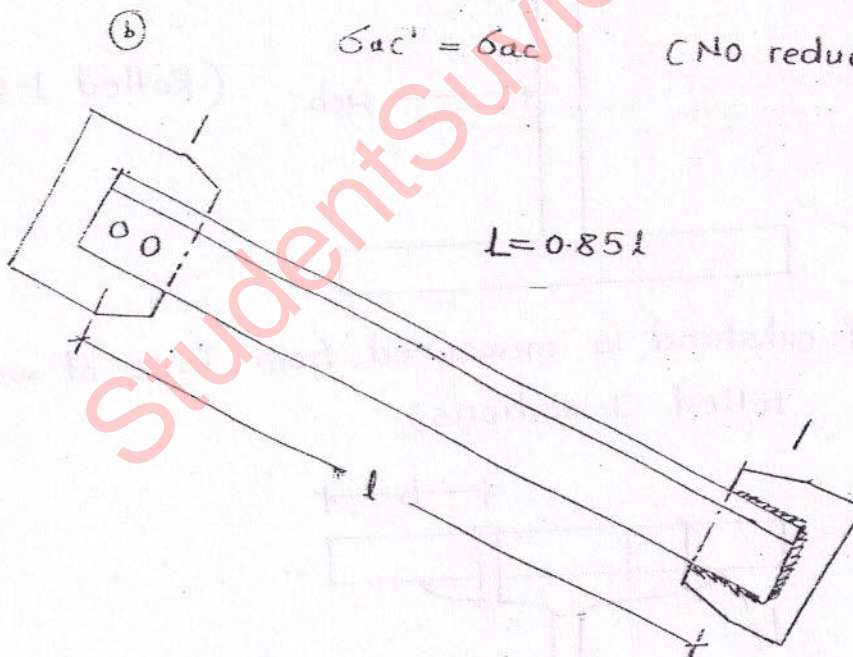
(iii) If a single angle discontinuous strut is connected by two or more rivets or weld then,

⊙ Effective length.

$$L = 0.85 l$$

$$\sigma_{ac} = \sigma_{ac}$$

(No reduction in σ_{ac})



* (iv) If there is possibility of out of plane buckling of strut i.e. if strut buckles out of plane of truss, then effective length is

$$L = l.$$

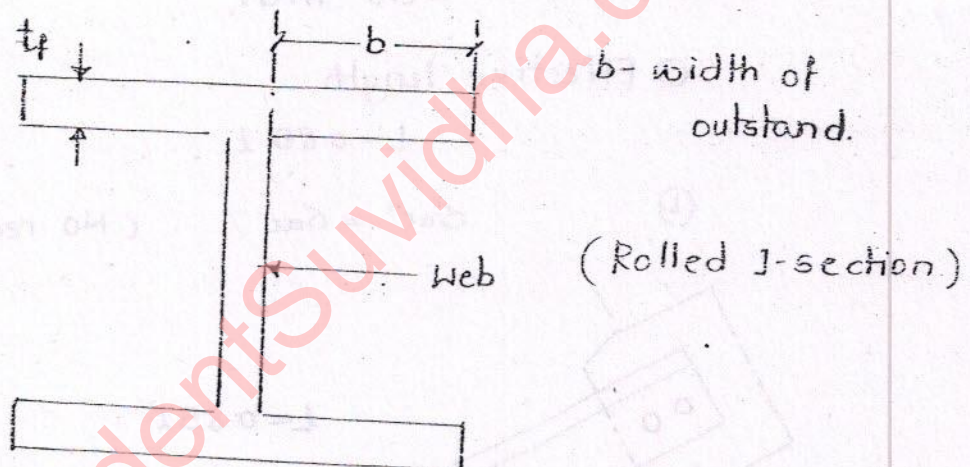
Local buckling of flange & web plates of I-section:

- a) When flange plate and web plate of I-section are subjected to compression, then they buckle locally. The critical stress by which the local buckling occurs is given as.

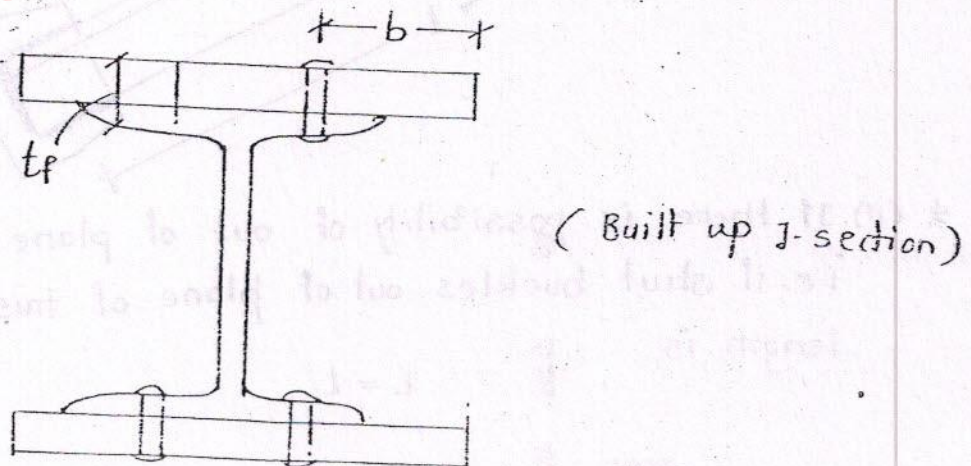
$$f_{cr} = \frac{k \cdot \pi^2 E}{12 (1 - \mu^2) \left(\frac{b}{t}\right)^2}$$

(from Theory of plates)

$$f_{cr} \propto \frac{1}{\left(\frac{b}{t}\right)^2}$$



(Width of outstand is measured from face of web for rolled I-sections)



(width of outstand is measured from outer line of rivet to extreme edge) for built up sections.

To prevent local buckling of flange plate and web plate due to compression IS 800 specifies the following provisions:

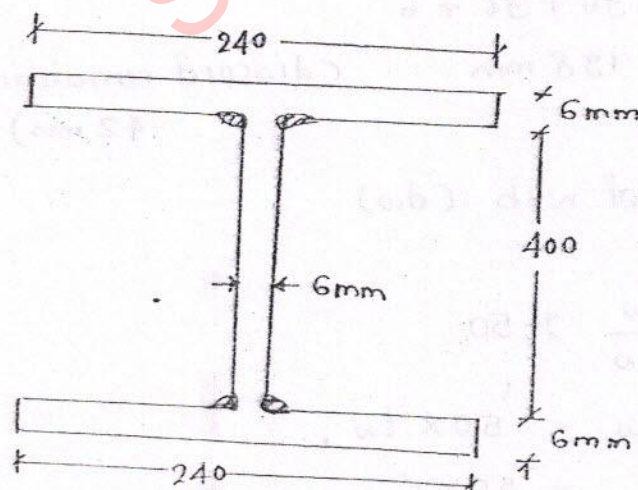
$$\frac{b_f}{t_f} \nless 16 \quad (\text{in WSM})$$

$$\frac{d_w}{t_w} \nless 50 \quad (\text{in WSM})$$

and $\frac{b_f}{t_f} \nless 8.4 \quad (\text{in LSM})$
(for plastic section)

If the flange and web plate dimension exceed the above limits, the excess width should be neglected while calculating moment of inertia, radius of gyration etc.

Q. Find the permissible compressive load on an axially loaded steel column having c/s area as shown in fig. Effective length of column is 3.5 m. (For purpose of calculating c/s area moment of inertia, radius of gyration, max. width of outstand $b_f \nless 16 t_f$ and max. depth of web $d_w \nless 50 t_w$)



λ	σ_{ac}
60	113
70	107.5
80	100.7
90	92.8
100	84

If it is a standard rolled steel section $\frac{b}{t_f}$, $\frac{b_w}{t_w}$ values need not be checked because the manufacturer's take care of above limitations.

1) Analysis :

Load carrying capacity of column

$$P_{safe} = \sigma_{ac} \times A_g$$

where,

A_g - effective gross c/s area of column.

σ_{ac} - permissible axial compression stress in column (depends on λ)

(a) Effective gross c/s area (A_g)

(i) Effective flange width (b_f)

for flanges,

$$\frac{b}{t_f} \geq 16$$

b - width of strand.

$$\begin{aligned} b &= 16 \times 6 \\ &= 96 \text{ mm} \end{aligned}$$

$$b_f = 96 + 96 + 6$$

$$= 198 \text{ mm} \quad (\text{discard remaining } 42 \text{ mm})$$

(ii) Effective depth of web (d_w)

for web,

$$\frac{d_w}{t_w} \geq 50$$

$$d_w = 50 \times t_w$$

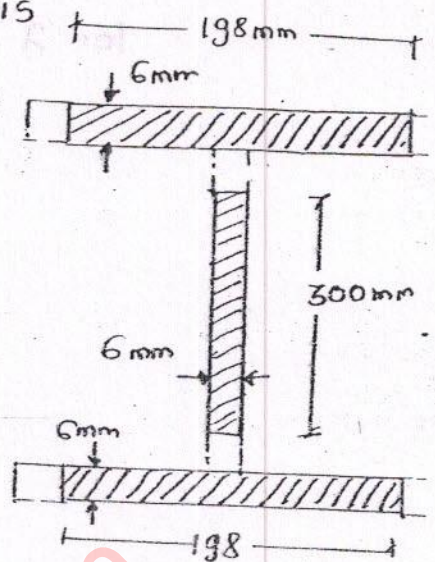
$$= 50 \times 6$$

$$= 300 \text{ mm}$$

(discard remaining 100 mm)

So, effective gross C/s of column is

$$A_g = (198 \times 6) \times 2 + (300 \times 6) \\ = 4176 \text{ mm}^2$$



⑥ To find σ_{ac}

σ_{ac} depends on $\lambda = \frac{L}{r_{min}}$

$$L = 3.5 \text{ m (given)}$$

$$r_{min} = \sqrt{\frac{I_{min}}{A_g}}$$

$$I_{xx} = \left[\frac{198 \times 6^3}{12} + (198 \times 6) (203)^2 \right] \times 2 + \left[\frac{6 \times 300^3}{12} \right] \quad \text{for web}$$

$$= 111.4 \times 10^6 \text{ mm}^4$$

$$I_{yy} = \left[\frac{6 \times 198^3}{12} \right] \times 2 + \left[\frac{300 \times 6^3}{12} \right]$$

$$= 7.768 \times 10^6 \text{ mm}^4$$

$$I_{min} = I_{yy} = 7.768 \times 10^6 \text{ mm}^4$$

$$r_{min} = \sqrt{\frac{7.768 \times 10^6}{4176}}$$

$$= 43.11 \text{ mm}$$

$$\lambda = \frac{L}{r_{min}}$$

$$= \frac{3500}{43.11}$$

$$= 81.11$$

from linear interpolation.

for $\lambda = 81.11$.

$$\begin{aligned}\sigma_{ac} &= 100.7 - \frac{1.11}{10} (100.7 - 92.8) \\ &= 99.7 \text{ N/mm}^2\end{aligned}$$

$$\begin{aligned}P_{safe} &= A_g \times \sigma_{ac} \\ &= 4176 \times 99.7 \\ &= 416.5 \text{ kN.}\end{aligned}$$

Q. An ISWB 300 is used as a column of height 6m fixed at base and hinged at top. Find the permissible compressive load on column.

For ISWB 300 - $A_g = 6133 \text{ mm}^2$

$$b_f = 200 \text{ mm} \quad t_f = 10 \text{ mm}$$

$$t_w = 7.4 \text{ mm} \quad (t_w \text{ is always less than } t_f)$$

$$I_{xx} = 98.216 \times 10^6 \text{ mm}^4$$

$$I_{yy} = 9.9 \times 10^6 \text{ mm}^4$$

λ	σ_{ac}
110	72
120	64

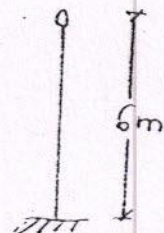
Since it is a standard section, $\frac{b}{t_f}$ and $\frac{d_w}{t_w}$ values need not be checked.

Analysis:

∴ Effective length, $L = 0.8l$

$$\begin{aligned}&= 0.8 \times 6000 \\ &= 4800 \text{ mm}\end{aligned}$$

$$\lambda_{min} = \sqrt{\frac{I_{min}}{A_g}}$$



$$= \sqrt{\frac{9.9 \times 10^6}{6133}}$$

$$= 40.18 \text{ mm}$$

$$\lambda = \frac{4800 \text{ mm}}{40.18}$$

$$= 119.46$$

For $\lambda = 119.46$.

$$\sigma_{ac} = 72 - \frac{9.46}{10} \times (72 - 64)$$

$$= 64.43 \text{ N/mm}^2$$

$$P_{safe} = A_g \times \sigma_{ac}$$

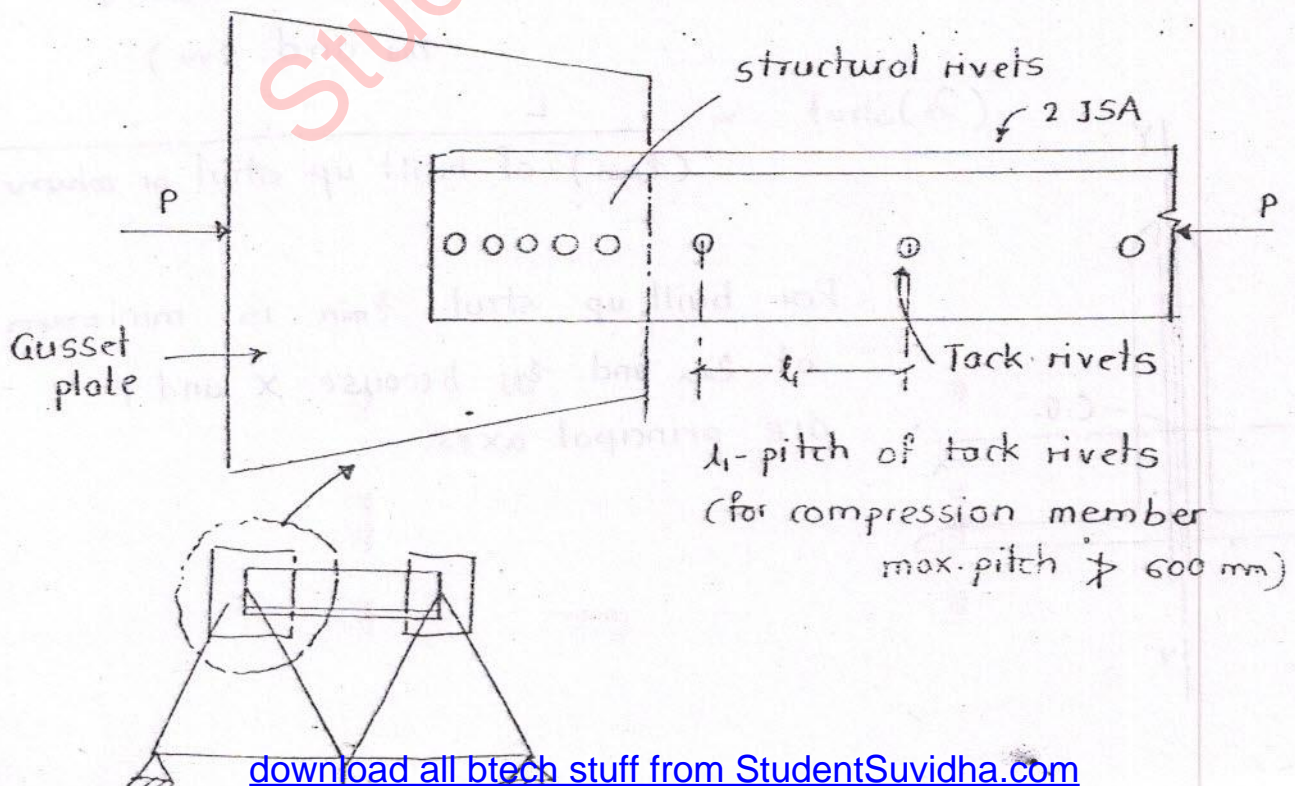
$$= 6133 \times 64.43$$

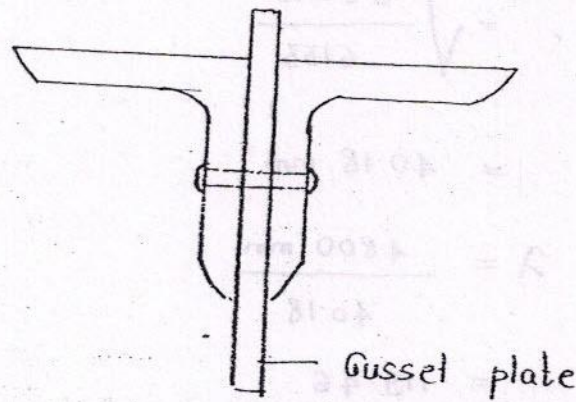
$$= 395.6 \text{ kN.}$$

Sunday

27th October 2013

Analysis of struts:





1/5 view

If tack rivets are used to connect components of the strut then there is possibility of buckling of strut component between strut tack rivets. To prevent buckling of component following condition should be satisfied.

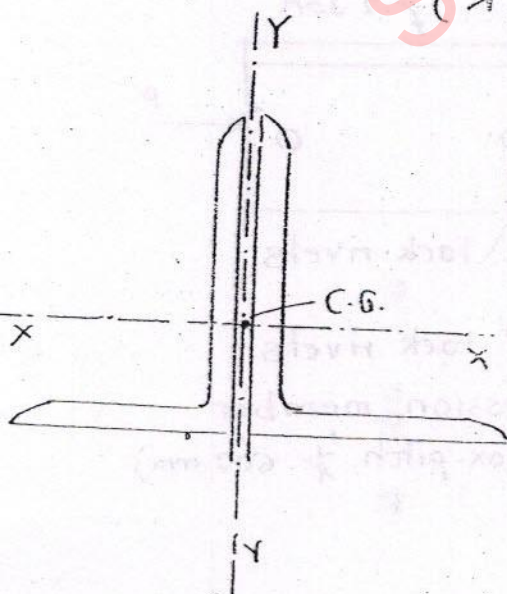
$$(\lambda)_{\text{component}} \neq 40$$

$$\neq 0.6 \lambda_{\text{strut}}$$

$$(\lambda)_{\text{component}} = \frac{l_1}{(\lambda_{\min})_{\text{component}}}$$

(λ_{\min} is minimum of λ_{xx} and λ_{yy})

$$(\lambda)_{\text{strut}} = \frac{L}{(\lambda_{\min}) \text{ of built up strut or column}}$$



For built up strut, λ_{\min} is minimum of λ_{xx} and λ_{yy} because X and Y are principal axes.

Q. A continuous principal rafter (main compression member in a truss) of a truss is 3m long between intermediate connections. It consists of 2-ISA - 90x90x8 mm on both sides of 16 mm gusset plate. The two angles are tack rivetted by using 16 mm dia. rivets at 30 cm intervals. Effective length is taken as (0.85 x distance between the intersection). Find the load carrying capacity of the rafter.

Given: ISA 90x90x8 mm

$$A_g = 1379 \text{ mm}^2$$

$$r_{xx} = r_{yy} = 27.5 \text{ mm}$$

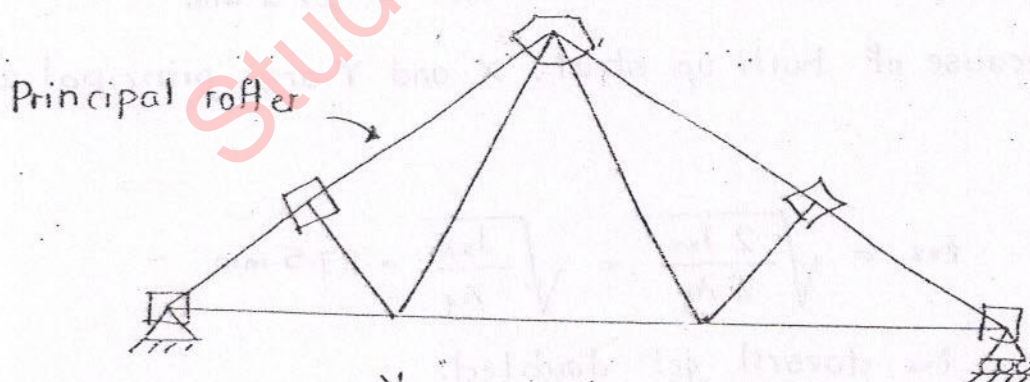
$$r_{uu} = 34.7 \text{ mm}$$

$$r_{vv} = 17.5 \text{ mm}$$

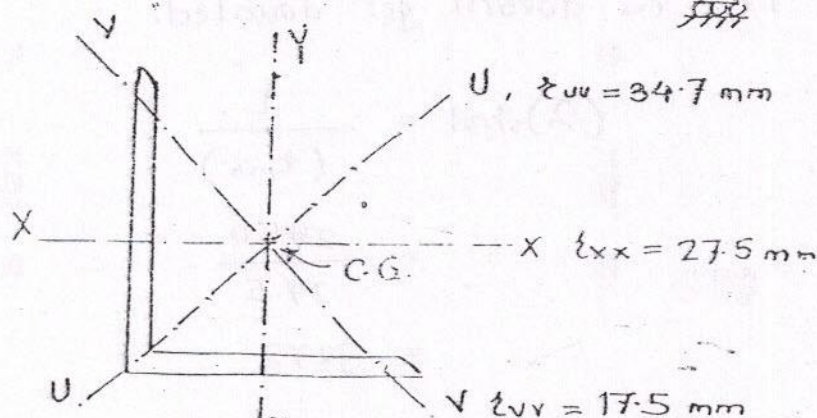
$C_{xx} = C_{yy}$ = Centroidal distance from back of angle = 25.1 mm.

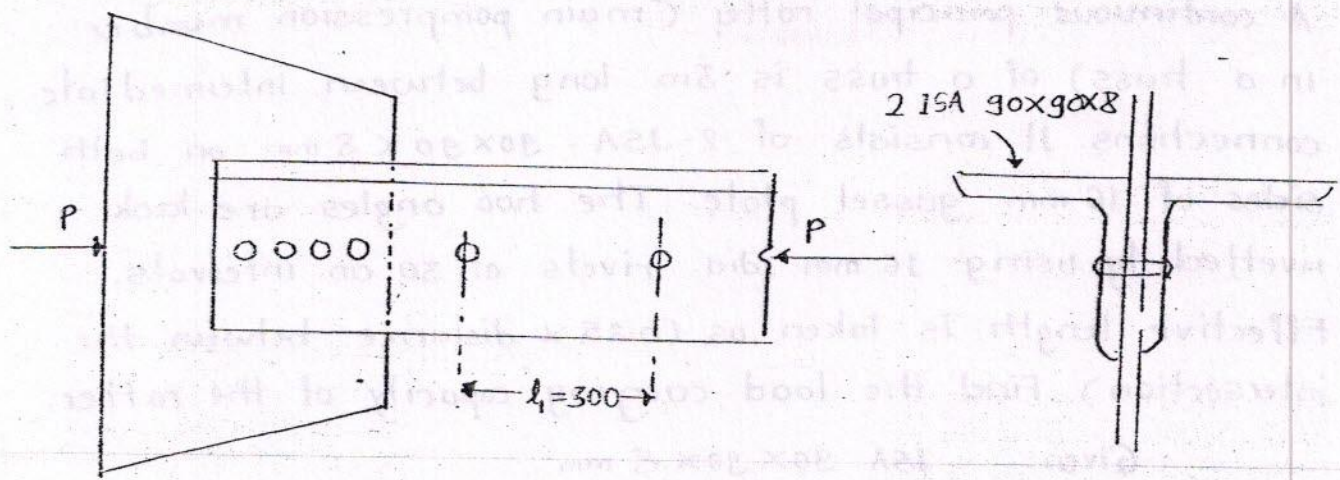
λ	σ_{ac}
70	112
90	90
110	72

Note:



X, Y - if two angles are used for a built up section.





$$P_{safe} = \sigma_{ac} \times A_g$$

A_g - gross effective area.

$$A_g = 2 \times 1379 \text{ mm}^2$$

σ_{ac} - permissible axial compressive stress depends on λ .

$$(\lambda)_{strut} = \frac{L}{(\lambda_{min})}$$

$$L - \text{effective length of strut} = 0.85 \times 3 \\ = 2.55 \text{ m}$$

$$\lambda_{min} - \text{min. of } \lambda_{xx} \text{ and } \lambda_{yy} = 27.5 \text{ mm}$$

(Because of built up strut, X and Y are principal axes)

Note:

$$\lambda_{xx} = \sqrt{\frac{2 I_{xx}}{2 A_g}} = \sqrt{\frac{I_{xx}}{A_g}} = 27.5 \text{ mm}$$

i.e. λ_{xx} doesn't get doubled.

$$\begin{aligned} (\lambda)_{strut} &= \frac{L}{(\lambda_{min})} \\ &= \frac{2550}{27.5} \\ &= 92.72 \end{aligned}$$

for $\lambda = 92.72$.

$$\begin{aligned}\sigma_{ac} &= 90 - \frac{2.72}{20} \times (90 - 72) \\ &= 87.5 \text{ N/mm}^2\end{aligned}$$

$$\begin{aligned}P_{safe} &= \sigma_{ac} \times A_g \\ &= 87.5 \times (2 \times 1379) \\ &= 241.32 \text{ kN.}\end{aligned}$$

Check for buckling of column components between lath rivets.

To prevent buckling,

$$\begin{aligned}(\lambda)_{\text{component}} &\nless 40 \\ &\nless 0.6 (\lambda)_{\text{strut}}\end{aligned}$$

$$\begin{aligned}(\lambda)_{\text{component}} &= \frac{L_1}{r_{\min}} \\ &= \frac{300}{17.5} \quad r_{\min} = r_{yy} = 17.5 \text{ mm} \\ &= 17.14 \quad \nless 40 \\ &\quad \nless 0.6 \times 92.72 = 55.63\end{aligned}$$

so buckling of components will not happen.

Design of compression member.

Load (P) is given, we have to fix the size of member.

Procedure:

- (i) Assume σ_{ac} (for rolled sections assume $\sigma_{ac} = 60 - 80 \text{ MPa}$ & for built up section assume $\sigma_{ac} = 110 - 120 \text{ MPa}$)

High value for built up sections because the buckling possibility is less. ($I_{xx} \nless I_{yy}$)

(ii) Find $(A_g)_{\text{required}}$

$$(A_g)_{\text{required}} = \frac{P}{\sigma_{ac}}$$

(iii) Select a suitable section from the given set of sections and find λ , σ_{ac} .

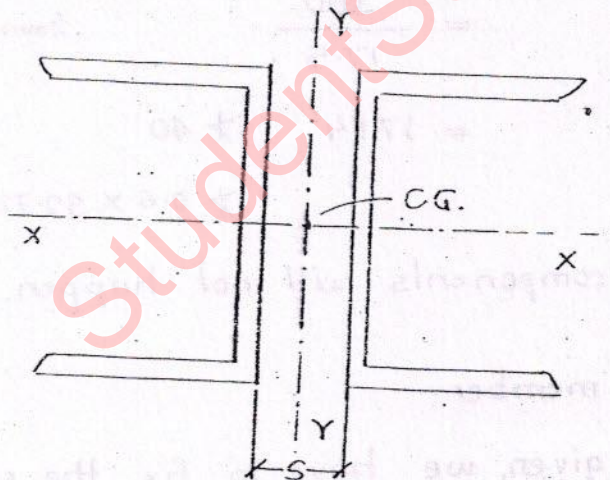
(iv) Find P_{safe} .

$$P_{\text{safe}} = \sigma_{ac} \times (A_g)_{\text{provided}} \geq P_{\text{applied}}$$

Then, design is safe.

Q. Two channels are placed back to back and used as a column. Spacing of columns channels is such that $J_{xx} \approx J_{yy}$. If spacing of channels is doubled, then load carrying capacity is --

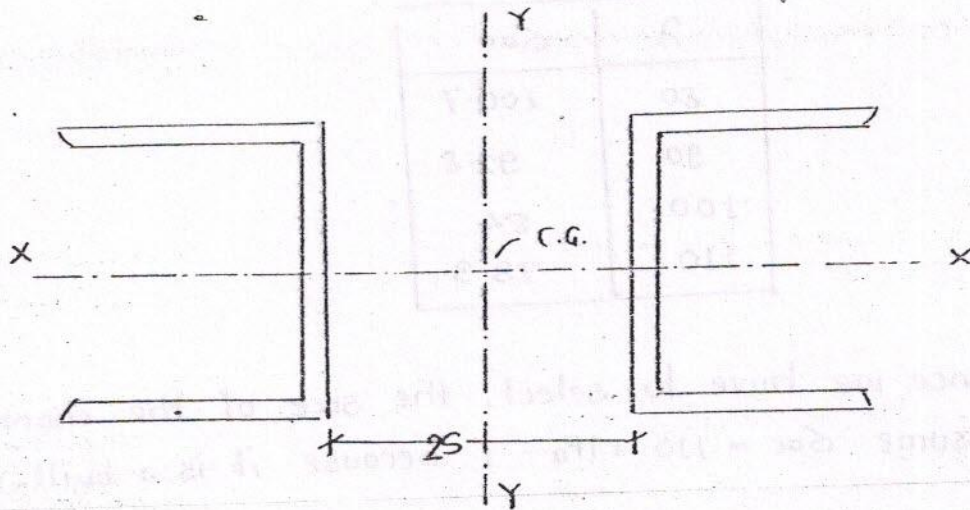
- (a) is doubled (b) is halved
(c) remains same (d) none.



$$J_{xx} = J_{yy}$$

$$P_{cr} = \frac{\pi^2 E J_{\min}}{L^2}$$

$$P_{cr} = \frac{\pi^2 E J_{xx}}{L^2}$$



J_{xx} remains same but J_{yy} increases (due to transfer formula)

$$P_{cr} = \frac{\pi^2 E J_{min}}{L^2} = \frac{\pi^2 E J_{xx}}{L^2}$$

$\therefore P_{cr}$ will not be changed if spacing is increased.

So while designing built column, adjust the column components such that $J_{xx} = J_{yy}$. Then column will be equally strong about both axis.

Q. Design of built up column of effective length 10m to carry an axial load of 750 kN. Use two channels placed back to back. Available channel sections are

ISMC 250

$$A_g = 3867 \text{ mm}^2$$

$$z_{xx} = 99.4 \text{ mm}$$

$$z_{yy} = 23.8 \text{ mm}$$

$$C_{yy} = 23 \text{ mm}$$

$$J_{xx} = 3816.8 \times 10^4 \text{ mm}^4$$

$$J_{yy} = 219.1 \times 10^4 \text{ mm}^4$$

ISMC 300

$$A_g = 4564 \text{ mm}^2$$

$$z_{xx} = 118.1 \text{ mm}$$

$$z_{yy} = 26.18 \text{ mm}$$

$$J_{xx} = 6362.6 \times 10^4 \text{ mm}^4$$

$$J_{yy} = 310.8 \times 10^4 \text{ mm}^4$$

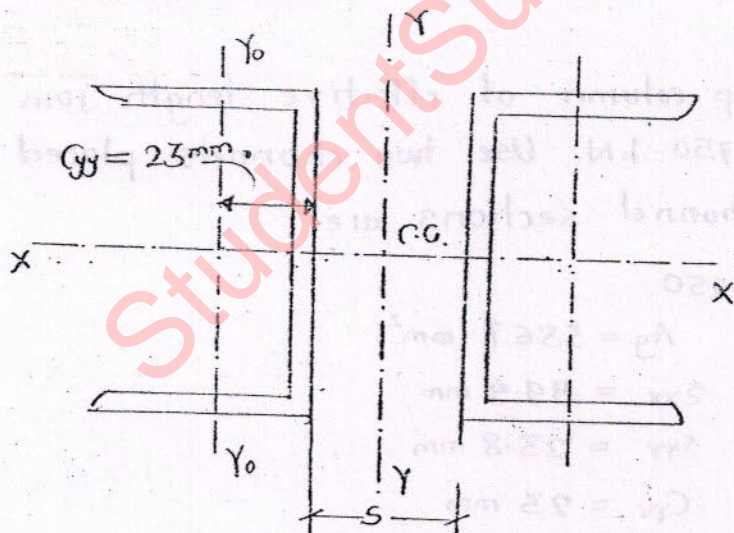
λ	σ_{ac}
80	100.7
90	92.8
100	84
110	73.3

- (i) Since we have to select, the size of the channel sections assume $\sigma_{ac} = 110 \text{ MPa}$ (because it is a built up column)

$$\begin{aligned} \therefore (A_g)_{\text{reqd}} &= \frac{P}{\sigma_{ac}} \\ &= \frac{750 \times 10^3 \text{ N}}{110 \text{ N/mm}^2} \\ &= 6818.8 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} \text{For each channel, } (A_g)_{\text{reqd}} &= \frac{6818.8}{2} \\ &= 3409.1 \text{ mm}^2 \end{aligned}$$

- (ii) Select 2-JSMC 250.



Provide spacing (s) such that $I_{xx} = I_{yy}$, so that, the column is equally strong about both axis.

Note:

By providing spacing (s), we have increased M.I. of individual channel sections from ($I_{yy} = 219.1 \times 10^4$) to $I_{yy} = 3816.8 \times 10^4 \text{ mm}^4$. It means that for built up column I_{yy} is increased. It mean that r_{yy} increased ($23.8 \text{ mm} \rightarrow 99.4$)

so, for built up column,

$$z_{xx} = z_{yy} = 99.4 \text{ mm.}$$

(iii)

$$P_{\text{safe}} = (\sigma_{ac}) \times A_g.$$

$$(A_g)_{\text{provided}} = 2 \times 3867$$

=

σ_{ac} depends on λ .

$$\lambda = \frac{L}{z_{\min}}$$

$$= \frac{10 \times 10^3 \text{ mm}}{99.4 \text{ mm}}$$

$$= 100.6 \text{ mm} \quad (z_{\min} \text{ not } 23.4 \text{ mm})$$

$$= 100.6 \text{ mm} < 180 \text{ mm} \quad (\text{i.e. max. permissible slenderness ratio})$$

Hence, safe.

For $\lambda = 100.6$

$$\sigma_{ac} = 84 - \frac{0.6}{10} (84 - 75.3)$$

$$= 83.5 \text{ N/mm}^2$$

$$P_{\text{safe}} = \sigma_{ac} \times (A_g)_{\text{provided}}$$

$$= 83.5 \times (2 \times 3867)$$

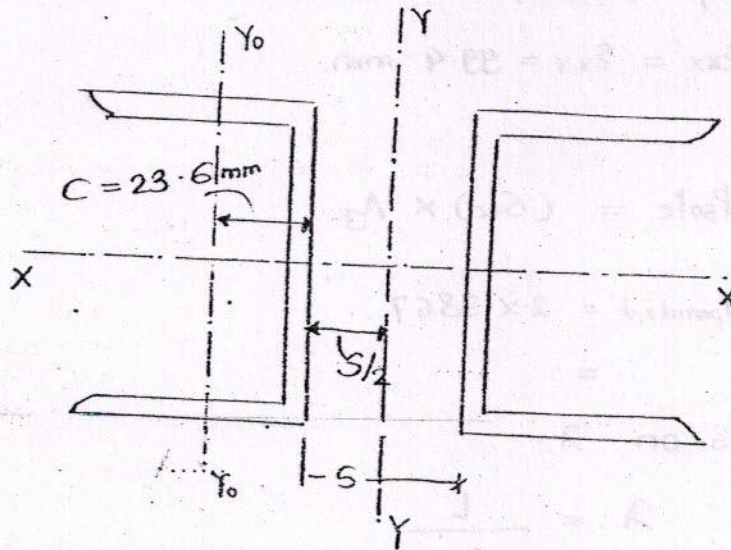
$$= 645.7 \text{ kN} < 750 \text{ kN}$$

Hence, unsafe.

Note:

Column become unsafe because, length of column is very high. So, chose two ISMC-300 and check for the load carrying capacity.

(ii) Selected 2-ISM C-300



$$\lambda = \frac{L}{r_{\min}}$$

$$= \frac{10 \times 10^3}{118.1}$$

$$= 84.67$$

r_{\min} is not 23.18 mm

For $\lambda = 84.67$,

$$\begin{aligned} \sigma_{ac} &= 100.7 - \frac{4.67}{10} (100.7 - 92.8) \\ &= 97.01 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} P_{\text{safe}} &= \sigma_{ac} \times A_g \\ &= 97.01 \times (2 \times 4564) \\ &= 885 \text{ kN} > 750 \text{ kN.} \end{aligned}$$

Hence design is safe.

(v) Spacing of channels (S)

$$I_{xx} = I_{yy}$$

$$\frac{2 (6362.6 \times 10^4)}{2 \text{ channels } I_{xx}} = \frac{2 \left[\frac{310.8 \times 10^4}{2 \text{ channels } I_{y_0 y_0}} + \frac{4564 \times \left(23.6 + \frac{S}{2} \right)^2}{A_g h^2} \right]}{2 \text{ channels } I_{y_0 y_0}}$$

$$S = 183.5 \text{ mm}$$

provide spacing of 185 mm.